Reg. No. :

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B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Fourth Semester

Mechanical Engineering (Sandwich)

MA 6453 – PROBABILITY AND QUEUEING THEORY

(Common to Computer Science and Engineering/Information Technology)

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. If $M_X(t) = \dfrac{pe^t}{1-qe^t}$ is the Moment Generating function of X then find the mean and variance of X.

2. The mean of Binomial distribution is 20 and standard deviation is 4. Find the parameters of this distribution.

3. Let (X, Y) be a continuous Bivariate Random Variable. If X and Y are independent Random Variables then show that X and Y are uncorrelated.

4. Define conditional distributions.

5. State any two properties of Poisson Process.

6. If the transition probability matrix of a Markov chain is $P = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$, find the stationary distribution of the chain.

7. What is the probability that a customer has to wait more than 15 minutes to get his service completed in $(M|M|1):(G_D/\infty/\infty)$ queueing system if $\lambda = 6$ per hour and $\mu = 10$ per hour ?

8. Define the following terms : Balking, Reneging and Jockeying.

9. Write the PollacZek-Khinchin formula for the case when the service time is constant.

10. Write an expression for traffic equation open Jackson Network.

PART – B **(5×16=80 Marks)**

11. a) i) Find the moment generating function of Gamma distribution and hence find its mean and variance. **(10)**

ii) The scores on an achievement test given to 1,00,000 students are normally distributed with mean 500 and standard deviation 100. What should the score of a student be to place him among the top 10% of all students ? **(6)**

(OR)

b) i) Determine the Moment Generating function of a normal random variable X with parameters (μ, σ^2), Hence find its mean and variance. **(8)**

ii) Let X be uniformly distributed random variable in the interval (a, 9) and $P[3 < x < 5] = 2/7$. Find the constant 'a' and compute $P[|x-5| < 2]$. **(8)**

12. a) The joint probability density function of the two dimensional random variable (X, Y) is given by $f(x, y) = \frac{x}{4}(1+3y^2), 0 < x < 2, 0 < y < 1$. Find

i) Conditional probability density functions of X given Y = y and Y given X = x. **(8)**

ii) $P[0.25 < X < 0.5 / Y = 0.33]$. **(8)**

(OR)

b) i) Given that X = 4Y + 5 and Y = kX + 4 are regression lines of X on Y and Y on X respectively. Show that $0 \le k \le \frac{1}{4}$. If $k = \frac{1}{16}$, find the means of X and Y and the correlation coefficient r_{XY}. **(8)**

ii) If the joint pdf of (X, Y) is given by $f(x, y) = x + y,\ 0 \le x, y \le 1$, find the pdf of the R.V. U = XY. **(8)**

13. a) i) If X(t) is wide sense stationary process with autocorrelation $R(\tau) = Ae^{-\alpha|\tau|}$, determine Second order moment of the random variable X(8) – X(5). **(8)**

ii) Show that random process {X(t)} where $X(t) = A\cos(\omega_0 t + \theta)$ is a wide sense Stationary process if A and ω_0 are constants and θ is uniformly distributed random variable over $(0, 2\pi)$. **(8)**

(OR)

b) i) Find the mean and auto correlation functions of Poisson Process. **(8)**

ii) Classify the states of the Markov chain for the one-step transition probability matrix $P = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$ with state space S = {1, 2, 3}. **(8)**

14. a) i) Derive the Steady state probabilities for (M | M | 1) : (GD/N/∞) queueing model and hence obtain the expressions for Ls and Ws. **(10)**

ii) A petrol pump station has 4 pumps. The service times follow the exponential disribution with a mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 car per hour. What is the probability that an arrival would have to wait in line ? **(6)**

(OR)

b) i) Derive the formulae for Lq and Wq for (M/M/C) : (GD/∞/∞), C >1 queueing model. **(8)**

ii) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

1) What is the probability that an arriving patient will not wait ?

2) What is the expected waiting time until a patient is discharged from the clinic ? **(8)**

15. a) i) For an $(M/E_2/1) : (FIFO/\infty/\infty)$ queueing model with $\lambda = \frac{6}{5}$ per hour and $\mu = \frac{3}{2}$ per hour, find the average waiting time of a customer. Also find the average time he spends in the system. **(8)**

ii) In a college canteen, it was observed that there is only one waiter who takes exactly 4 minutes to serve a cup of coffee once the order has been placed with him. If students arrive in the canteen at an average rate of 10 per hour, how much time one is expected to spend waiting for his turn to place the order. **(8)**

(OR)

b) i) Obtain the PollacZek-Khinchin formula for the $(M/G/1) : (G_D/\infty/\infty)$ queueing model. **(10)**

ii) A repair facility shared by a large number of machines has 2 sequential stations with respective service rates of 2 per hour and 3 per hour. The cumulative failure rate of all the machines is 1 per hour. Assuming that the system behaviour may be approximated by the 2-stage tandem queue, find the average number of machines in service at both the stations and find the average repair time including the waiting time. **(6)**